Effectiveness of electron-cyclotron and transmission resonance heating in inductively coupled plasmas

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The electron-cyclotron and transmission resonances in magnetically enhanced low-pressure one-dimensional uniform inductively coupled plasmas are studied analytically within a simple model of two driven electrodes. The results of our approach are also applicable to the case of one grounded electrode. It is shown that, for a high discharge frequency, the plasma resistance is greatly enhanced at electron-cyclotron and transmission resonances, but normally does not exhibit a sharp peak at the electron-cyclotron resonance (ECR) condition. For a low discharge frequency, the ECR heating is not effective. Conditions of strong transmission resonances are identified. A transition from a bounded to semi-infinite plasma with overlapping of transmission resonances is also considered. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2034407]

Low-pressure, radio-frequency inductively coupled plasmas (ICP) find applications in semiconductor manufacturing and lighting.[1] The need for optimization of ICP discharges has prompted intensive research on basic plasma phenomena in collisionless plasmas, as described in reviews.[2–6]

Operating a magnetically enhanced ICP with a magnetic field near or above the electron-cyclotron resonance (ECR) condition $\Omega_c \geqslant \omega$ (where $\Omega_c = eB/mc$ is the electron-cyclotron frequency and ω is the discharge frequency) can lead to a considerable increase in power coupling due to efficient electron-wave interaction for $\Omega_c \approx \omega$ and enhanced rf field penetration into the plasma for $\Omega_c > \omega$.[1,2,7–11] Measurements of plasma characteristics of magnetized ICP discharges presented in Refs. 9 and 11 show the growth of power coupling and plasma surface resistance with increasing magnetic field, until the discharge becomes unstable.[11] However, a sharp maximum in the plasma surface resistance at the ECR condition, typical of plasma heating in the gigahertz range of frequencies, was not observed in the megahertz range for commonly employed discharge parameters.[11] In this work, the efficiency of electron-cyclotron and transmission resonant heating is investigated using a kinetic warm plasma approach for uniformly bounded and semi-infinite plasmas.

The inductive rf electric field driven by an external rf current $I\exp(-i\omega t)$ can be determined from Faraday's and Ampère's laws. Their combination gives the amplitude of the rf field $E_y \exp(-i\omega t)$

$$\frac{d^2E_y}{dx^2} + \frac{\omega^2}{c^2}E_y = -\frac{4\pi i\omega}{c^2}[j(x) + I\delta(x) - I\delta(x-L)]. \qquad (1)$$

Here, I is the amplitude of the current at plasma boundaries ($x=0$ and $x=L$). The current at $x=L$ is flowing in the opposite direction to that at $x=0$ (shifted in phase by π). The one-dimensional slab geometry system of two surface currents flowing in opposite directions provides a good description of a solenoidal discharge with diameter $D=L$ and also describes approximately a "pancake" geometry with one coil at $x=0$ and a grounded electrode at $x=L/2$ (corresponding to the boundary condition $E_y=0$ at $x=L/2$).[10,12,13] If the effective mean free path $\lambda_{\mathrm{eff}} = V_T/\sqrt{(\omega-\Omega_c)^2+\nu^2}$ is small compared with the discharge gap $\lambda_{\mathrm{eff}} \ll L$, then two antennas act independently and the total deposited power into the plasma can be viewed as the sum of two halves, which are the same as for one antenna at one plasma side and for the grounded electrode ($E=0$) at $x=L/2$. In Fig. 1 the plasma surface resistance is calculated using the formalism of Ref. 14 for one grounded electrode with plasma length $L/2$ and utilizing the much simpler formalism of the two driving antennas with plasma length L for typical plasma parameters. Apparently, the agreement between the two cases is very good, if not excellent. In this article, we chose the one-dimensional slab geometry of two surface currents because the analytical solution in this case is much simpler and easier to analyze, while the results are similar to the just-mentioned configurations for typical plasma parameters.

In the presence of an external static magnetic field perpendicular to the plasma boundary, the rf electric field splits into left-polarized ($E^l = E_z + iE_y$) and right-polarized ($E^r = E_z - iE_y$) waves. The electron conductivity for each Fourier component of the electric field is given by

$$j_k^{l,r} = \frac{e^2 n}{im|k|V_T} Z_M\left(\frac{\omega \pm \Omega_c + i\nu}{|k|V_T}\right) E_k^{l,r}, \qquad (2)$$

where $Z_M(\xi)$ is the plasma dispersion function see Ref. 15. The substitution of $\omega \to \omega \pm \Omega_c$ accounts for the electron gyration in the magnetic field (see Ref. 2) The electric-field

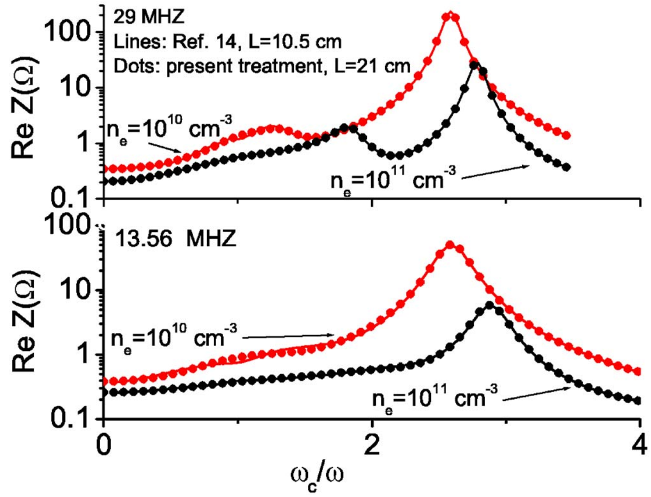

FIG. 1. (Color online). Surface resistance as a function of a normalized electron-cyclotron frequency. The solid lines correspond to the formalism of Ref. 14 for one grounded electrode with plasma length $L/2$ and the dotted lines correspond to the case of two driven electrodes with plasma length L. Shown: rf driving frequencies 29 and 13.56 MHz, electron temperature T_e =4 eV, and electron collision frequency $\nu=1.2\times10^7$ s^{-1}.

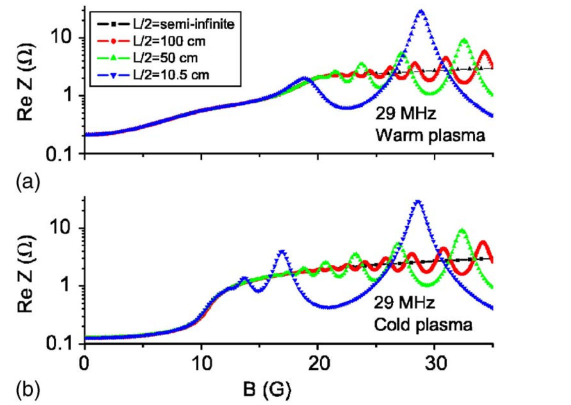

FIG. 2. (Color online). Surface resistance of semi-infinite and bounded plasmas of different lengths as a function of the applied magnetic field for a uniform plasma with a Maxwellian EEDF. The discharge frequency is 29 MHz, the plasma density $n_e=10^{11}$ cm^{-3}, and the electron collision frequency $\nu=1.2\times10^7$ s^{-1}. (a) Warm plasma with the electron temperature T_e=4 eV and (b) cold plasma with the electron temperature $T_e\to 0$ (local approximation for electron current).

profile is obtained by applying the inverse Fourier transform method to Eq. (1),

$$E^{l,r}(x)=\frac{8\pi i\omega I}{c^2L}\times\sum_{s=0}^{\infty}\frac{\cos(k_sx)}{k_s^2-\omega^2/c^2-Z_M[(\omega\pm\Omega_c+i\nu)/(k_sV_T)]/k_s\delta_a^3}. \quad (3)$$

Here, s is an integer, $k_s=(2s+1)\pi/L$, and we introduced the anomalous skin depth $\delta_a=[c^2V_T/(\omega_{\rm pe}^2\omega)]^{1/3}$, where $\omega_{\rm pe}=(4\pi e^2n/m)^{1/2}$ is the electron plasma frequency. The surface impedance is given by the ratio of the electric field to the rf magnetic field or applied current at the plasma boundary $\mathbb{Z}=(-2E/I)|_{x=0}$.

The surface impedance of a one-dimensional, bounded, uniform plasma of length L, with a Maxwellian electron energy distribution function (EEDF), inductively driven by two current sheets with an applied external static magnetic field is $\mathbb{Z}=(\mathbb{Z}^l+\mathbb{Z}^r)/2$ (see Ref. 13), where

$$\mathbb{Z}^{l,r}=\frac{16\pi i\omega}{c^2L}\sum_{s=0}^{\infty}\frac{1}{k_s^2-\omega^2/c^2-Z_M[(\omega\pm\Omega_c+i\nu)/(k_sV_T)]/k_s\delta_a^3}. \quad (4)$$

In the limit of a semi-infinite uniform plasma, $L\to\infty$, the summation turns into an integral with $dk\to2\pi/L$ and Eq. (4) yields

$$\mathbb{Z}_\infty^{l,r}=\frac{8i\omega}{c^2}\int_0^\infty dk\frac{1}{k^2-\omega^2/c^2-Z_M[(\omega\pm\Omega_c+i\nu)/(kV_T)]/k\delta_a^3}. \quad (5)$$

The power P deposited in the plasma per unit area is related to the real part of the surface impedance $R=\mathrm{Re}(\mathbb{Z})$ (surface resistance) as $P=2I^2R$.

At the relatively high frequency of 29 MHz, the plasma skin effect is normal—the skin depth of width $c/\omega_{\rm pe}$ is larger than the nonlocality length V_T/ω. Under these conditions the power dissipation without an applied magnetic field is small and depends on both collisional and collisionless effects [cf. Figs. 2(a) and 2(b)]. The plasma surface resistance initially increases with magnetic field as the cyclotron frequency approaches the wave frequency. Note that all curves for different plasma lengths coincide in the region below the ECR condition ($B<B_c\simeq10$ G, where $B_c=mc\omega/e$ is the magnetic field corresponding to ECR), because in all cases the skin depth is much smaller than the plasma half-length and the two skin layers on the opposite plasma boundaries are independent of each other ($\lambda_{\rm eff}\ll L$). It should be noted that the surface resistance does not exhibit a sharp maximum at the electron-cyclotron resonance condition ($B_c\simeq10$ G for 29 MHz), and this differs strikingly from the case of interaction of a magnetized electron with a prescribed rf wave. At the exact condition of the electron-cyclotron resonance $\Omega_c=\omega$, the surface resistance of a collisionless plasma $\nu\ll\omega$, with a plasma slab length much larger than the anomalous skin depth δ_a, can be calculated from Eq. (5) [taking into account that at ECR $\mathrm{Re}\,Z_M(0)=0$ and $\mathrm{Im}\,Z_M(0)=\sqrt{\pi}$ (see Ref. 16)]. For the right-hand polarized wave it yields

$$Z_{\rm ECR}^r=\frac{8\pi^{5/6}}{3}\left(i+\frac{1}{\sqrt{3}}\right)\frac{\omega\delta_a}{c^2}. \quad (6)$$

Equation (6) predicts a larger plasma surface resistance at the electron-cyclotron resonance with increasing rf field frequency ω and rf field penetration depth (see also Ref. 10).

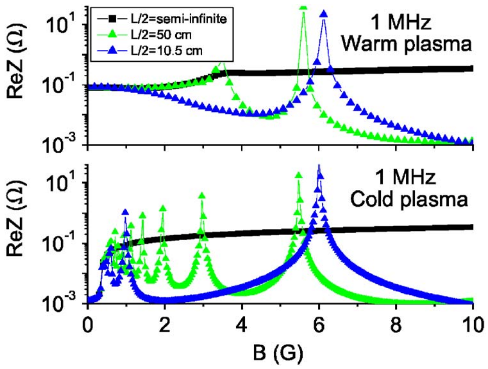

FIG. 3. (Color online). Surface resistance of semi-infinite and bounded plasmas of different lengths as a function of the applied magnetic field for a uniform plasma with a Maxwellian EEDF. The discharge frequency is 1 MHz, the plasma density $n_e=10^{11}$ cm^{-3}, and the electron collision frequency $\nu=1.2\times10^5$ s^{-1}. (a) Warm plasma with the electron temperature $T_e=4$ eV and (b) cold plasma with the electron temperature of $T_e\to0$ (local approximation for electron current).

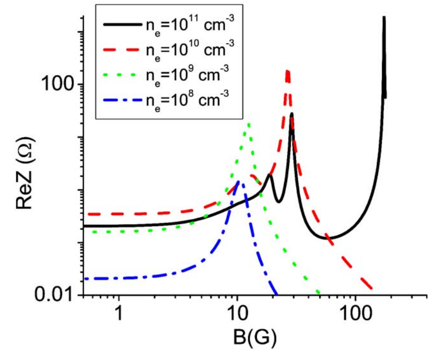

FIG. 4. (Color online). Surface resistance of warm bounded uniform Maxwellian plasmas for different electron densities as a function of the applied magnetic field. Discharge parameters: rf field frequency $\omega/2\pi=29$ MHz, electron temperature $T_e=4$ eV, electron collision frequency $\nu=1.2\times10^7$ s^{-1}, and the plasma half-length $L/2=10.5$ cm.

The latter can occur either due to an increase in electron temperature or a decrease in plasma density. Under the ECR condition ($B_c\simeq10$ G) the electron interaction with the rf electric field is similar to heating in a dc electric field and the nonlocality length increases up to the mean free path V_T/ν, determined by the collision frequency ν. The plasma surface resistance at the ECR condition, $\omega-\Omega_c\ll V_T/\delta_a$, is identical to the plasma surface resistance at the condition of the anomalous skin effect, $\omega\ll V_T/\delta_a$. Consequently, for low discharge frequencies, for which the condition of the anomalous skin effect is satisfied, the application of a magnetic field does not enhance the plasma surface resistance at the ECR, as shown in Fig. 3, where the ECR condition occurs at $B=0.3$ G.

Increasing the external magnetic field above the ECR condition ($B\geqslant B_c\simeq10$ G for 29 MHz) leads to the further growth of the plasma surface resistance, as evident in Figs. 2 and 3. This is due to propagation of the right-hand polarized wave into the plasma see Refs. 7, 8, and 10. Analysis of the wave propagation is especially convenient in the cold plasma approximation. In the limit of high magnetic field, warm plasma effects are not important if $\Omega_c-\omega\gg V_Tk_s$. Substituting the cold plasma limit of the dielectric function $Z_M(\zeta)\to-\zeta^{-1}$ for $\zeta\to\infty$ gives the poles of the electric field in Eq. (3) as $c^2k_p^2=\omega^2-\omega_{\rm pe}^2/(1\pm\Omega_c/\omega)$. For a typical magnetically enhanced ICP $\omega,\Omega_c\ll\omega_{\rm pe}$ and propagating modes exist only for the right-hand polarized wave with a wave vector $k_p=\omega_{\rm pe}/(c\sqrt{\Omega_c/\omega-1})$. Therefore, the conditions of existence of transmission resonances are

$$\frac{V_T}{\Omega_c-\omega}\ll\frac{L}{(2s+1)\pi}, \tag{7}$$

$$\frac{(2s+1)\pi}{L}=\frac{\omega_{\rm pe}}{c\sqrt{\Omega_c/\omega-1}}. \tag{8}$$

Equation (7) shows that warm plasma effects can be neglected for not very high resonance numbers s, for which the nonlocality length is less than the wavelength, and Eq. (8) shows that for a bounded plasma, transmission resonance occurs if an odd number of half-waves equals the plasma slab length $k_p=k_s$ (see also Ref. 10, where similar conditions were obtained). Strong transmission resonances at the values of the magnetic field predicted by Eq. (8) are evident in Figs. 2 and 3. Note that the transmission resonances occur at different values of the magnetic field for different plasma slab lengths. When Eq. (7) is satisfied, the surface resistance of cold plasma is the same as that of warm plasma [cf. Figs. 2(a) and 2(b)]. In the opposite case, transmission resonances are less pronounced due to wave damping [cf. Figs. 3(a) and 3(b)]. The maximum value of the plasma surface resistance and the width of the transmission resonances are determined by a small dissipation, either due to collisional or collisionless damping described by $\mathrm{Im}(Z_M)$. Note that a right-hand polarized wave is reflected from a plasma-vacuum interface with a reflection coefficient $R=1-2\omega/ck_p$. Since $\omega/ck_p\ll1$, $R\simeq1$ and the wave is trapped inside the plasma.

Let us now estimate the condition on plasma parameters for the existence of transmission resonances. Substitution of $\Omega_c-\omega$ from Eq. (8) into Eq. (7) yields

$$L\gg\pi\delta_a, \tag{9}$$

or the plasma length L must be larger than the anomalous skin depth δ_a. For short plasmas or low plasma densities Eq. (9) is not satisfied and the transmission resonances are not observed as it is shown in Fig. 4 for the electron density $n_e<10^{10}$ cm^{-3}. As a result, the plasma surface resistance decreases with increasing applied magnetic field $B>B_c$. In addition, Eq. (9) gives the maximum value of the wave vector k_p for a pronounced transmission resonance $k_p\ll1/\delta_a$. That is, the wavelength should be much longer than the anomalous skin effect length. This condition provides that the collisionless damping of the wave is small. Substitution of k_s from Eq. (8) into Eq. (7) results in

$$\Omega_c-\omega\gg(V_T\omega_{\rm pe}\omega^{1/2}/c)^{2/3}, \tag{10}$$

which gives the minimum value of $\Omega_c-\omega$ for a pronounced transmission resonance for a given plasma density. Note that for a fixed plasma length, Eq. (10) is applicable only for plasma densities satisfying Eq. (9). The huge variation of the surface resistance at strong transmission resonances may cause difficulty in coupling power to the plasma through a matching network, as was reported in Ref. 11 for magnetic fields $B > 20$ G.

As evident in Figs. 2 and 3 increasing the length of a bounded plasma leads to a larger number of transmission resonance peaks. For large L, these peaks overlap and the plasma surface resistance of a bounded plasma reaches its asymptotic curve given by the surface resistance of the semi-infinite plasma. Substituting the cold plasma limit for the dielectric function $Z_M(\zeta) \to -\zeta^{-1}$ as $\zeta \to \infty$ and integrating Eq. (5) over the poles of the electric field, $k=k_p$, yield the asymptotic value of the plasma surface resistance for large magnetic fields $B > B_c$,

$$\mathbb{Z}_{\infty}^{r} = \frac{4\pi}{c\omega_{\mathrm{pe}}}\sqrt{\omega(\Omega_c - \omega)}. \tag{11}$$

The condition on plasma length for applicability of the limit of a semi-infinite plasma is given by

$$L > L_{\max} = \frac{4\pi k_p^2 \delta_a^3}{\mathrm{Im}\, Z_M[(\omega \pm \Omega_c + i\nu)/(k_p V_T)]}. \tag{12}$$

It can be derived taking into account that for overlapping of the transmission resonances, the resonance width $\delta k = \mathrm{Im}\, Z_M[(\omega \pm \Omega_c + i\nu)/(k_p V_T)]/(2k_p^2\delta_a^3)$ must be larger than the distance between them $dk=2\pi/L$, or $\delta k > dk$, as it follows from Eq. (4). This condition corresponds to strong dumping of the propagating wave on distance $L_{\max}$ (see Ref. 18).

In conclusion, application of a static magnetic field can considerably enhance the plasma surface resistance and efficiency of power deposition under the conditions of electron-cyclotron and transmission resonances (see Refs. 10, 17, and 18). The plasma surface resistance at the ECR condition is similar to that of a very low discharge frequency without an applied magnetic field described by the theory of anomalous skin effect $(\omega - \Omega_c \ll V_T/\delta_a)$. For short and not very dense plasmas, when $L < \pi\delta_a$, the maximum of plasma resistance occurs at the ECR condition. For sufficiently long and dense plasmas, when $L > \pi\delta_a$, the plasma surface resistance is higher for $eB/mc > \omega$ due to a deeper wave penetration into the plasma and the maximum resistance occurs at transmission resonances due to standing wave or cavity effects.

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