SIMULTANEOUS CONSTRAINED MODEL PREDICTIVE CONTROL AND IDENTIFICATION OF DARX PROCESSES

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ABSTRACT

In this work, we formulate a new approach to simultaneous constrained Model Predictive Control and Identification (MPCI). The proposed approach relies on the development of a persistent excitation (PE) criterion for processes described by DARX models. That PE criterion is used as an additional constraint in the standard on-line optimization of MPC. The resulting on-line optimization problem of MPCI is handled by successively solving a series of semi-definite programming problems. Advantages of MPCI in comparison to other closed-loop identification methods are (a) Constraints on process inputs and outputs are handled explicitly, (b) Deterioration of output regulation is kept to a minimum, while closed-loop identification is performed. The applicability of the method is illustrated by a number of simulation studies. Theoretical and computational issues for further investigation are suggested.

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1. INTRODUCTION

Constrained Model Predictive Control (MPC) relies on a model of the controlled process to select optimal process inputs by performing a constrained on-line optimization at each time step (Prett and García, 1988). The process model is usually made available at the controller design stage, after conducting open-loop tests on the process. The performance of a model predictive controller depends on the employed model. Performance may become unacceptable due to a very inaccurate model. Of course, a robust controller can tolerate a certain amount of model inaccuracy. A number of methodologies have recently emerged that address the rigorous design of robustly stable constrained model predictive controllers (Genceli and Nikolaou, 1993; Vuthandam *et al*., 1995; Zheng and Morari, 1994; Michalska and Mayne, 1993). In all methodologies, the closed-loop performance would be better if the model represented the process as closely as possible, and at all times. In addition, if closed-loop performance of a given MPC feedback system were not acceptable, re-identification of the process model under feedback control might be desirable. Therefore, a need exists for the development of closed-loop identification techniques for constrained MPC.

Over the past several years closed-loop identification of linear systems, in general, has been studied and analyzed in great detail by various authors. A good review of early results on identifiability and accuracy aspects of identification under closed-loop conditions can be found in Gustavsson *et. al.* (1977). The effect of different feedback configurations on system identifiability has been treated by Gustavsson *et al.* (1977); Ljung *et. al*. (1974); Söderström *et. al*. (1975). Box and MacGregor (1975) studied closed-loop identification in a stochastic framework. Wellstead and Edmunds (1975) give sufficient conditions for uniqueness and consistency of parameter estimates using least-squares identification under closed-loop conditions. Feldbaum (1965) introduced the idea of dual control to simultaneously identify and control a process. Unfortunately, dual control was not practical due to the complicated dependence of the control action on parameter estimates. Hence, the *certainty equivalence principle* was used to design sub-optimal dual controllers (Goodwin and Payne, 1977; Wieslander and Wittenmark, 1971; Tse *et. al*. 1973; Mehra, 1974). Alster and Belanger (1974) attempted to develop a closed-loop identification approach based on constraining the trace of the information matrix to be positive. Unfortunately, this condition does not even guarantee invertibility of the information matrix.

The recent years have seen a resurgence of research on closed-loop identification and controller design. Many authors (Schrama, 1992a; 1992b; Gevers, 1991; Voda and Landau, 1995; Bitmead 1993) have presented an iterative approach to joint closed-loop identification and controller design. Koung and MacGregor (1994) presented an approach to the design of experiments for control relevant identification of multivariable processes. All these approaches, rely on some kind of experimental design that employs an external dithering signal for the purpose of identification. Fu and Sastry (1991) gave a direct design of optimal inputs in the frequency domain for on-line identification and model reference adaptive control. All of the above approaches do not explicitly handle constraints on process inputs and outputs. In addition, their primary concern is closed-loop identification of the process, while process regulation performance is either not addressed, or is of secondary importance.

Genceli and Nikolaou (1995) developed a new class of algorithms to closed-loop identification and adaptive control. Their approach, referred to as simultaneous model predictive control and identification (MPCI) answers the following question:

How can a model be re-identified for a process under constrained MPC, with minimum loss of regulation performance and satisfaction of process input/output constraints?

MPCI resorts to on-line optimization. At each time step, MPCI minimizes a rather conventional quadratic objective function over a finite moving horizon. The minimization is performed with respect to process inputs that satisfy a strong persistent excitation (PE) constraint, in addition to all conventional MPC constraints. MPCI has an element of *duality* in it: learning and regulation. For learning, it ensures that inputs to the process are exciting enough to yield information about the process dynamics. With regard to regulation, the process input tries to keep the process output at a desired setpoint. The PE condition, the main distinguishing feature between MPCI and MPC, can be turned on and off, depending upon the identification needs of the closed-loop. The advantage of MPCI is that it does not require any external dithering signal, and the closed-loop performance deteriorates minimally while the process is identified. In addition, constraints on process inputs are treated explicitly, and constraints on process outputs are explicitly handled in the best possible way. Moreover, parameter convergence is easy to establish, at least for finite-impulse-response models. A locally optimal solution of the on-line optimization problem can be obtained at each time step by successively solving a series of semidefinite programming (SDP) problems (Genceli and Nikolaou, 1995).

In the work by Genceli and Nikolaou (1995), MPCI was developed for processes described by deterministic moving-average (DMA) models. MPCI for DMA models gives rise to on-line optimization problems of high dimensionality. In this work, we extend MPCI to deterministic auto-regressive moving average (DARX) processes in an effort to reduce the dimensionality of the on-line optimization problem solved by MPCI. The difficulty lies in that the PE condition for DARX models depends on the process output (Goodwin and Sin, 1984), a signal on which the designer has no control. In the sequel, we develop a strong PE condition for DARX models that involves only process inputs. This PE condition is then used in the formulation of an on-line optimization problem that purports to select optimal process inputs for simultaneous identification and control of the process.

The rest of this paper is structured as follows: First, we present a theorem which guarantees a strong PE condition for DARX processes, independent of the process output. We then use this new PE condition in the formulation of MPCI with DARX models. We subsequently discuss the numerical solution of the resulting on-line optimization problem. Finally, we illustrate the above ideas through simulations.

2. PERSISTENT EXCITATION

The role of PE in system identification has been emphasized by several authors, including Ljung (1987); Goodwin and Sin (1984); Goodwin and Payne (1977); Anderson and Johnson (1982); Bitmead (1984); Lozano and Zhao (1994); Genceli and Nikolaou (1995). Parameter identification is feasible if the input to the process satisfies the PE property so as to excite all the modes of the system (Ljung, 1987). It has been shown (Anderson and Johnstone, 1983; Johnstone *et al*., 1982; Bittanti *et. al*.1990) that PE guarantees exponential parameter convergence for time-invariant plants. Also, it is well known (Anderson, 1985) that absence of PE in closed-loop identification for adaptive control gives rise to *bursting*, a phenomenon in which the output exhibits unstable or highly oscillatory behavior in between quiescent periods as a result of noise.

For DMA models the standard PE condition involves process inputs only (Goodwin and Sin, 1984). For DARX models the standard PE condition involves both process inputs and outputs. This prevents the standard PE condition from being easily usable within an MPCI framework. A PE condition involving process inputs only is desirable. In this section we formulate a PE condition involving process inputs only. That PE condition guarantees the standard PE condition, if a bound on the process model order is known. The resulting PE condition can be used in MPCI on-line optimization.

Consider a single-input, single output time-invariant process modeled in the DARX form by

$$y(k) = \sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{m} b_i u(k-i) + d + \varepsilon(k) \tag{1}$$

where $A(q)=1-a_1q^{-1}-a_2q^{-2}-\cdots-a_nq^{-n}$, and $B(q)=b_1q^{-1}+b_2q^{-2}+\cdots+b_mq^{-m}$ are coprime polynomials, d is a constant disturbance and ε denotes noise with zero mean. Let

$$\theta_o=\left[a_1\ a_2\ \cdots a_n\ b_1\ b_2\ \cdots\ b_m\ d\right]^T \tag{2}$$

be the parameter vector to be identified. Equation (1) can be written in the standard regression form as

$$y(k)=\phi(k-1)^T\theta_o+\varepsilon(k) \tag{3}$$

where $\phi(k-1)=\left[y(k-1)\quad\cdots\quad y(k-n)\quad u(k-1)\quad\cdots\quad u(k-m)\quad 1\right]^T$ is the regression vector. The estimate $\overline{\theta}$ of θ_o can be obtained by standard least-square error minimization (Goodwin and Sin, 1984). A unique solution is guaranteed if the information matrix $\sum_{i=1}^{s+1}\phi(k-i)\phi(k-i)^T$ is invertible and bounded (Goodwin and Sin, 1984). This condition is implied by the following strong PE condition:

$$\Phi\ \hat{=}\sum_{i=1}^{s+1}\phi(k-i)\phi(k-i)^T\preceq\rho_2\mathbf{I}\prec\infty \tag{4a}$$

$$\Phi\ \hat{=}\sum_{i=1}^{s+1}\phi(k-i)\phi(k-i)^T\preceq\rho_1\mathbf{I}\succ\mathbf{0} \tag{4b}$$

The upper limit, $\rho_2<\infty$ implies that the information matrix, Φ, is bounded. The existence of ρ_2 in the above inequality (4a) for stable processes with bounded manipulated inputs is trivially guaranteed, owing to the boundedness of the entries of the matrix Φ. The lower limit, $\rho_1 > 0$, in inequality (4b) implies that the information matrix is nonsingular.

It should be noted that the matrix Φ in inequality (4b) is a function of both the process inputs and the process outputs for a process modeled by a DARX model, as noted above. If we want to design inputs that guarantee equation (4b), then the PE condition should be reformulated in terms of the inputs only, independently of the process outputs. We thus need to find a condition that guarantees the inequality (4b) and that involves only the process manipulated inputs. In the following theorem, we develop such a condition that ensures the inequality (4b).

2.1 Theorem

The invertibility of the matrix

$$\Psi=\sum_{i=1}^{p+1}\psi(k-i)\psi(k-i)^T \tag{5}$$

guarantees the invertibility of the information matrix

$$\Phi = \sum_{i=1}^{s+1} \phi(k-i)\phi(k-i)^T \tag{6}$$

where

$$\psi(k-i) = \begin{bmatrix} u(k-i) & \cdots & u(k-i-n-m+1) & 1 \end{bmatrix}^T ,$$

$$\phi(k-i) = \begin{bmatrix} y(k-i-1) & \cdots & y(k-i-n) \; \vdots \; u(k-i-1) & \cdots & u(k-i-m) & 1 \end{bmatrix}^T$$

and $p \leq s-n$.

Proof: See Shouche (1996).

***Remark* :** For the matrix Ψ to be invertible, it must be $p \geq m+n$.

2.2 Reformulating PE

The previous Theorem implies that

$$\Psi \hat{=} \sum_{i=1}^{p+1} \psi(k-i)\psi(k-i)^T \succeq \gamma_1 \mathbf{I} \succ \mathbf{0} \tag{7}$$

$$\Rightarrow \Phi \hat{=} \sum_{i=1}^{s+1} \phi(k-i)\phi(k-i)^T \succeq \rho_1 \mathbf{I} \succ \mathbf{0}$$

Therefore, the PE condition involving the matrix Φ in equation (8) is guaranteed, if the above inequality (7) involving the matrix Ψ is satisfied. The advantage of using the inequality involving the matrix Ψ is that the entries of Ψ are functions of the inputs *u* only. Satisfaction of the inequality (7) implies that the process input *u* is strongly persistently exciting of order $m+n+1$. While this condition can be applied in the off-line design of persistently exciting inputs for identification, it can also be used on-line for simultaneous identification and control of a process. In the next section, we explain how to design inputs which will simultaneously identify and control a process.

3. MODEL PREDICTIVE CONTROL AND IDENTIFICATION (MPCI)

3.1 Formulation of MPCI

Let the following linear time-varying model be used to represent the process at time *k*.

$$y(l|k) = \sum_{i=1}^{n} a_i(l|k) y(l-i|k) + \sum_{i=1}^{m} b_i(l|k) u(l-i|k) + d(l|k) = \phi(l \mid k)^T \theta(l \mid k) \tag{8}$$

where:

$$y(l|k) = \begin{cases} \text{current or past measured output }, y(l), \text{ at time } l \le k \\ \text{future predicted output at time } l > k \end{cases}$$

$$u(l|k) = \begin{cases} \text{implemented process input}, u(l), \text{ at time } l < k \\ \text{current or future potential input at time } l \ge k \end{cases}$$

$d(l|k)$ = disturbance estimate at time l made at time k

$a(l|k)$ = model coefficient estimate at time l made at time k

$b(l|k)$ = model coefficient estimate at time l made at time k

$$\phi(l|k) = \begin{bmatrix} y(l|k-1) & \cdots & y(l|k-n) & u(l|k-1) & \cdots & u(l|k-m) & 1 \end{bmatrix}^T \tag{9}$$

$$\theta(l|k) = \begin{bmatrix} a_1(l|k) & \cdots & a_n(l|k) & b_1(l|k) & \cdots & b_m(l|k) & 1 \end{bmatrix}^T \tag{10}$$

Applying equation (8) for $l = k-s+1, \cdots k$ and assuming that

$$\theta(k \mid k) \approx \theta(k-1 \mid k) \approx \cdots \approx \theta(k-s+1 \mid k) \tag{11}$$

we can use least squares to estimate the process parameters as

$$\bar{\theta}(k|k) = \arg \min_{\theta} \left(\begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-s+1) \end{bmatrix} - \begin{bmatrix} \phi(k \mid k)^T \\ \phi(k-1 \mid k)^T \\ \vdots \\ \phi(k-s+1 \mid k)^T \end{bmatrix} \theta \right)^T \begin{bmatrix} 1 & & & \\ & \lambda & & \\ & & \ddots & \\ & & & \lambda^{s-1} \end{bmatrix} \left(\begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-s+1) \end{bmatrix} - \begin{bmatrix} \phi(k \mid k)^T \\ \phi(k-1 \mid k)^T \\ \vdots \\ \phi(k-s+1 \mid k)^T \end{bmatrix} \theta \right) \tag{12}$$

where $0 \le \lambda < 1$. The *certainty equivalence principle* assumes that the parameters estimated at time k from equation (12) are the true system parameters over a finite horizon in the future. The following on-line optimization problem can then be set up for simultaneous identification and control:

MPCI Problem P_1

$$\min_{u(k|k),u(k+1|k),\cdots,u(k+M-1|k),\mu} \sum_{i=1}^{M} \left[w_i (y(k+i|k) - y^{sp})^2 + r_i \Delta u(k+i-1|k)^2 \right] + q\mu^2 \tag{13}$$

subject to

$$u_{\max} \ge u(k+i-1|k) \ge u_{\min}, i = 1,2,\cdots M \tag{14}$$

$$\Delta u_{\max} \ge \Delta u(k+i-1|k) \ge \Delta u_{\min}, i = 1,2,\cdots M \tag{15}$$

$$\sum_{j=0}^{M-1} \lambda^j \psi(k-j+i \mid k) \psi(k-j+i \mid k)^T \succeq (\gamma_1 - \mu) \mathbf{I} \succ \mathbf{0}, \quad i = 1,2,\cdots M \tag{16}$$

$$y(k+i|k) = \phi(k+i|k)^T \bar{\theta}(k|k) \tag{17}$$

$$\bar{\theta}(k|k) = \left[\sum_{j=0}^{s-1} \lambda^j \phi(k-j|k)\phi(k-j|k)^T\right]^{-1} \left[\phi(k|k) \quad \phi(k-1|k) \quad \cdots \quad \phi(k-s+1|k)\right] Y(k) \tag{18}$$

where

$$\Delta u(k+i-1|k) \hat{=} u(k+i-1|k) - u(k+i-2|k)$$

$$Y(k) = [y(k) \;\; y(k-1) \;\; \cdots \;\; y(k-s+1)]^T$$

$$\psi(k-j+i|k) = \left[u(k-j+i-1|k) \;\; u(k-j+i-2|k) \cdots u(k-j+i-m-n|k) \;\; 1\right]^T$$

$$\phi(k-j|k) = \left[y(k-j-1) \quad \cdots \quad y(k-j-n) \quad u(k-j-1) \quad \cdots \quad u(k-j-m) \quad 1\right]^T$$

$$M \geq m+n+1$$

$$s \geq 2n+m+1$$

In typical MPC fashion (Prett and Garcia, 1988), the above optimization problem is solved at time k, and the optimal $u(k)$ is applied to the process. This procedure is repeated at subsequent times k+1, k+2, etc.

Remarks

- The tuning parameters for MPCI are: 1)The Control horizon length, M; 2) The move suppression coefficients, r_i ; 3) the output weighting, w_i; 4) The identification window, s; 5) The excitation level, γ_1; 6) The relaxation weighting, q; 7) The forgetting factor, λ .
- The dual action of the controller is clear from equations (13) through (18). Indeed, the control objective, equation (13), is to minimize the cost corresponding to the error and the input moves over a horizon, while the PE constraint, equation (16), ensures that the input u is rich enough to generate sufficient information about the process. These two requirements are conflicting, hence the potential input moves achieve an optimal compromise between regulation and identification.
- The PE condition, equation (16), is a non-convex constraint in the form of quadratic matrix inequalities (QMI). The relative simplicity of equation (16) over the standard PE constraint, equation (4b) is apparent.
- If the inequality constraint in equation (16) is removed form the above MPCI optimization problem, equations (13)-(18), then the resulting on-line optimization problem corresponds to standard MPC with adaptation.
- The variable μ is a relaxation variable, which is used to guarantee a feasible solution for the on-line optimization. Ideally, μ should be 0.

- In contrast to MPC, for which the control horizon is usually shorter than the prediction horizon, the MPCI formulation involves a prediction horizon that has the same length, *M*, as the control horizon.
- Theorem 2.1 and equation (7) guarantee unique parameter estimates from equation (18) in the presence of the strong PE constraint given by equation (16)
- In conventional MPC, process parameter changes are lumped with unmeasured disturbances and the process model is not updated. In MPCI, however, both the process parameters as well as the disturbance are estimated on-line.
- For identification, a moving window of fixed length, *s*, is used by discarding an old data point for each new one added. The identification window should be large for a good projected signal to noise ratio (SNR). However, the assumption that the process is *quasi* time-invariant for a large identification window may not be true. This indicates that the selection of the identification window depends on the noise level and the rate of change of the process parameters. The level of excitation can be increased by increasing γ_1, although it should be noted that the highest possible excitation depends on the physical constraints on the process inputs. In the presence of noise, the excitation level should be greater than the sensor threshold to give a good signal to noise ratio. In section 4, we have included some simulations to show that for a given sensor threshold, there exists an optimal excitation level which will give best closed-loop performance.

3.2 Approximation of QMI *by* LMI

The PE, equation (16), is a non-convex constraint, as it is quadratic in the process input, *u*. To circumvent that problem, we take advantage of the inequality (Genceli and Nikolaou, 1995)

$$\sum_{j=0}^{M-1} \lambda^j \psi(k-j+i\,|\,k)\psi(k-j+i\,|\,k)^T \succeq \mathbf{L}_i(\mathbf{u}(k)) \quad i=1,2,\cdots M \tag{19}$$

where

$$\begin{aligned}\mathbf{L}_i(\mathbf{u}(k)) \hat{=} &\sum_{j=0}^{M-1} \lambda^j \psi^*(k-j+i\,|\,k)\psi(k-j+i\,|\,k)^T + \sum_{j=0}^{M-1} \lambda^j \psi(k-j+i\,|\,k)\psi^*(k-j+i\,|\,k)^T \\ &-\sum_{j=0}^{M-1} \lambda^j \psi^*(k-j+i\,|\,k)\psi^*(k-j+i\,|\,k)^T \;\forall i=1,2,\cdots,M\end{aligned} \tag{20}$$

$$\mathbf{u}(k) \hat{=} [u(k\,|\,k)\;\; u(k+1\,|\,k)\;\cdots\; u(k+M\text{ -}1\,|\,k)]$$

Therefore, by virtue of eqns. (19) and (20), the QMI in equation (16) are satisfied if the LMI

$$\mathbf{L}_i(\mathbf{u}(k)) \succeq (\gamma_1 - \mu)\mathbf{I} \quad i = 1,2,\cdots M \tag{21}$$

are satisfied. The vectors $\psi^* \in \Re^{n+m+1}$ in equation (20) can be thought of as points of linearization (Genceli and Nikolaou, 1995). It can further be shown (Genceli and Nikolaou ,1995) that the M LMI in equation (21) can be written as a single LMI.

The above discussion has shown that satisfaction of the LMI in equation (21) guarantees the satisfaction of the QMI in equation (16). Therefore, the optimal solution of the **MPCI optimization problem P_2**, comprised of equations (13), (14), (15), (21), (17), (18), will be a feasible but suboptimal solution of the original MPCI optimization problem $P_{1.}$ In the next subsection we discuss how the optimization problem P_2 can be formulated as a semidefinite programming problem.

***Remark*:** In the above formulation, the feasibility of the LMI constraint in eqn. (21) is always ensured because of the relaxation variable μ. Here we only use the same relaxation variable to relax all of the linearized PE constraints that are stacked diagonally in a single LMI. This formulation could over-relax all PE constraints because of just one PE constraint that might need to be relaxed. In lieu of this, one could relax each linearized PE constraint in the horizon $i = 1,\cdots,M$ with corresponding relaxation variables μ_i. The disadvantage of this approach is that the dimensionality of the optimization problem is increased.

3.3 MPCI for DARX processes as a Semi-definite Programming problem.

The problem P_2, as defined in the previous subsection, has the form of the following convex optimization:

$$\min_{\mathbf{x}} (\mathbf{Ex} + \mathbf{f})^T (\mathbf{Ex} + \mathbf{f}) \tag{22}$$

subject to

$$\mathbf{Ax} \geq \mathbf{b} \tag{23}$$

$$\mathbf{F}_o + \sum_{i=1}^{M+1} x_i \mathbf{F}_i \succeq \mathbf{0} \tag{24}$$

where

$$\mathbf{x} \triangleq [u(k \mid k) \quad u(k+1 \mid k) \quad \cdots \quad u(k+M-1 \mid k) \quad \mu]^T = [\mathbf{u}(k) \quad \mu]^T$$

Note that in contrast to the on-line optimization performed by MPC, the above optimization is not a *quadratic programming* (QP) problem, due to the additional LMI constraint, equation (24). It can be shown (Vandenberghe and Boyd, 1994) that the problem P_2 can be cast as a semidefinite programming (SDP) problem of the form

$$\min_{\mathbf{z}} \ \mathbf{c}^T \mathbf{z} \tag{25}$$

subject to

$$\mathbf{G}(z) = \mathbf{G}_0 + \sum_{i=1}^{q} z_i \mathbf{G}_i \succeq \mathbf{0} \tag{26}$$

where $\mathbf{z} \in \Re^q$, and $\mathbf{G}_0, \cdots, \mathbf{G}_q \in \Re^{r \times r}$ are symmetric matrices. For details see Shouche (1996).

3.4 Solution of the MPCI related SDP problem

The solution of the on-line SDP problem equivalent to P_2 depends on the selection of the linearization vectors ψ^*. Poorly selected ψ^* may result in a solution which is far from that of the original non-convex optimization problem P_1 (Genceli and Nikolaou, 1995). A successive SDP algorithm for the iterative selection of ψ^* and the on-line implementation of MPCI is proposed below. The algorithm has guaranteed convergence to a local optimum (Shouche 1996).

3.5 MPCI Algorithm

Step 1. Select $\left\{\psi^*(k-j+i|k), j = 0, \cdots, M-1, i = 1, \cdots, M-1\right\}$. A good choice is:

$$\psi^*(k-j+i|k) = \begin{bmatrix}\alpha_1 & \cdots & \alpha_{m+n} & 1\end{bmatrix}^T$$

where

$$\alpha_l = \begin{cases} u(k-j+i-l) \text{ if } & j-i+l \geq 1 \text{ (past } u\text{)} \\ u_{SDP}(k-j+i-l \mid k-1) \text{ if } & j-i+l < 1 \text{ (SDP solution at } k\text{ -1)} \end{cases}$$

Step 2. Solve the SDP problem, equivalent to P_2 (see Section 3.2). Let the optimal solution be

$$\{u_{SDP}(k \mid k), \quad \cdots, \quad u_{SDP}(k+M-1 \mid k), \quad \mu_{SDP}\}$$

Step 3. Update ψ^* as follows:

$$\psi^*_{new}(k-j+i|k) = \begin{bmatrix}\delta_1 & \cdots & \delta_{m+n} & 1\end{bmatrix}^T$$

where:

$$\delta_l = \begin{cases} u(k-j+i-l) \text{ if } & j-i+l \geq 1 \text{ (past } u\text{)} \\ u_{SDP}(k-j+i-l \mid k) \text{ if } & j-i+l < 1 \text{ (solution at } k\text{)} \end{cases}$$

Step 4. If $\left\|\psi^*_{new} - \psi^*_{old}\right\| \geq \kappa$ then let $\psi^*_{new} \leftarrow \psi^*_{old}$ and go to *Step 2*. Else go to *Step 5*.

Step 5. Implement the first input move $u_{SDP}(k \mid k)$ to the process.

Step 6. Identify the process parameter using equation (18).

Step 7. Let $k \leftarrow k+1$ and go to *Step 1*.

4. SIMULATION STUDIES

4.1 Case Study A: Process with a drifting parameter

Consider a linear process described by the following equation:

$$y(k) = 0.5u(k-1) + 0.06u(k-2) + b(k)y(k-1) - 0.3y(k-2) + d(k) \tag{27}$$

where the parameter b is drifting slowly as follows:

$$b(k) = 0.001k + 0.2 \quad 0 \leq k \leq 200 \tag{28}$$

The desired set-point is 0.1, and the process is upset by the following constant disturbance:

$$d(k) = \begin{cases} 0 & k \leq 10 \\ -0.05 & k > 10 \end{cases} \tag{29}$$

The process input has the following constraints on it:

$$\begin{aligned} -3.0 \leq u(k) \leq 3.0 \\ -2.0 \leq \Delta u(k) \leq 2.0 \end{aligned} \tag{30}$$

Let the process model employed by MPCI have the following form:

$$y(k+i \mid k) = a_1 u(k+i-1 \mid k) + a_2 u(k+i-2 \mid k) + b_1 y(k+i-1 \mid k) + b_2 y(k+i-2 \mid k) + d_1(k+i \mid k) \tag{31}$$

where the parameters $\left(a_1, a_2, b_1, b_2, d_1\right)$ are updated on-line.

Table 1 summarizes the tuning parameters used to simultaneously control and identify the process. Fig 1 shows the process input and output which have sufficient information to identify the slowly time varying parameter (b). This identified parameter is shown in Fig. 12. The process output, as expected, oscillates around the setpoint corresponding to the minimum excitation level (γ_1=0.1).

4.2 Case Study B: Effect of constraints

In this case study we examine the effect of input constraints on the quality of closed-loop identification. The process is described by

$$y(k) = ay(k-1) + bu(k-1) + d(k) \tag{32}$$

with values of a and b given in Table 2. The model being identified is of the form

$$y(k) = \hat{a}y(k-1) + \hat{b}u(k-1) + \hat{d}(k)\,. \tag{33}$$

The initial model considered at time k=0 is

$$y(k) = 0.4y(k-1) + 0.4u(k-1) \tag{34}$$

We compare three scenarios described in Table 2. The setpoint to the system is 0.2. The MPCI tuning parameters have the following values: M = 5, r = 0.2, n = 3, γ_1 = 0.02. Results of the simulation studies are presented in figures 3 – 8. Output and input mean values and mean squared output and input deviations for cases I and II are given in Table 3. The simulations reveal case I to produce no offset in its mean output value from the setpoint. Case II, however, shows an offset in the mean output value. This is because the MPCI controller in case II anticipates encountering input constraints. This causes the average value of resulting inputs to be lowered, to ensure the inputs have the desire PE. Also, an increase in the output and input deviations is observed in case II, where constraints come into effect. In case III, we observe in Figure 7 that the output is initially around the setpoint. But subsequently the gain of the system drifts and becomes smaller, thus requiring a larger input to drive the system to the setpoint. It is here again that the setpoints come into play and an offset can be observed.

4.6. Case Study C: Continuous stirred tank reactor

Consider a continuous stirred tank reactor (CSTR) with a cooling jacket. An exothermic reaction $A \to B$ is assumed to occur in the CSTR. The coolant temperature T_c is the manipulated variable to control the reactor temperature T. The dynamic behavior of the reactor is assumed to be described by the following ordinary differential equations

$$\frac{dC_A(t)}{dt} = \frac{F}{V}\left(C_{Ai} - C_A(t)\right) - C_A(t)ke^{-E/RT(t)}\,, \tag{35}$$

$$\frac{dT(t)}{dt} = \frac{F}{V}\left(T_i - T(t)\right) + \frac{(-\Delta H)}{\rho C_p}ke^{-E/RT(t)} - \frac{UA_t}{\rho C_p}(T(t) - T_c)\,. \tag{36}$$

where C_{Ai} is the inlet concentration, C_A is the concentration in the CSTR and the exit stream, T_i is the inlet temperature, T is the temperature in the CSTR and the exit stream, V is the volume of the CSTR, F is the flowrate, U is the overall heat transfer coefficient of the jacketed cooling system, A_t is the heat transfer area, ρ is the density, C_p is the heat capacity, and (-ΔH) is the heat of the reaction. The values of the parameters, inlet conditions, and the steady state conditions corresponding to those inlet conditions are given in Table 4. The following linear dynamic model was identified (using standard open-loop identification techniques) between the exit temperature and the coolant temperature.

$$\overline{T}(k) = 0.1589\overline{T}_c(k-1) + 0.3977\overline{T}(k-1) + 0.2545\overline{T}(k-2)\ , \tag{37}$$

where $\overline{T}$ and $\overline{T}_c$ are the deviations of the exit temperature and the coolant temperature from their steady state values. The control objective is to maintain the exit temperature of the outlet stream at the steady state value T_s, subject to the following constraints on the manipulated variable

$$\begin{aligned} -10.0 &\leq \overline{T}_c \leq 10.0, \\ -10.0 &\leq \Delta\overline{T}_c \leq 10.0. \end{aligned} \tag{38}$$

A change in the heat transfer coefficient changes the dynamic behavior of the process. The heat transfer coefficient is assumed to change to the value $UA_t = 6.0\times10^6$ ($J/K/h$) at time $t = 0$. Hence, in order to maintain the performance of the closed-loop, the process model has to be updated on-line without shutting down the process. The following methodologies were applied to update the process model while the process was controlled by an MPC scheme:

MPCI: The MPCI algorithm of Section 3.5 was applied. The process output was assumed to be corrupted with measurement noise. Table 5 summarizes the various parameters used in the MPCI scheme. The process input and the process output are shown in figures 9 and 10 (solid lines), respectively. It is clear from these figures that the controller performed the dual roles of identification and regulation. For identification, it perturbed the coolant temperature sufficiently to produce information about the process dynamics. For control, it kept the perturbations around the setpoint to a minimal level.

MPC with added external dithering signal to the coolant temperature: In this case, an external pseudo random binary sequence (PRBS) was added to the control signal coming from the model predictive controller. The magnitude of the PRBS signal was chosen to be $\sqrt{\gamma_1}$. The process input and the process output for this case study are shown in figures 9 and 10 (dashed lines) respectively. It is clear from these plots that the external dithering signal perturbed the process to produce information about its dynamic behavior. The parameters were identified using the same RLS algorithm as in the MPCI case.

Adaptive MPC without any external dithering: Here no excitation was provided for the purpose of identification. The input to the process coming from the controller, and the process output are sampled and used in the recursive least squares (RLS) routine.

The case studies done by applying MPCI and MPC with external dithering gave good parameter estimates due to the process excitation by the controller in MPCI, and the external dithering signal in MPC. However, the case study of MPC with no excitation gave very poor parameter estimates due to the lack of any information about the dynamics of the process, as illustrated in Figure 11.

Since the intended use of the updated model is to control the process, the performance of the closed-loop should be compared (using the different models obtained) when say a disturbance enters the process. Figure 12 shows such a comparison when a step disturbance (step change in the inlet concentration C_{Ai}) enters the process. For the cases when the model is not updated, and when the update is brought about without any excitation (adaptive MPC), the disturbance rejection of the controller is very poor. The cases when the model is updated using MPCI and using external dithering show good disturbance rejection properties. In fact, both MPCI and MPC with dithering show very close disturbance attenuation capabilities. However, it should be noted that the model updated using MPCI decreased the control performance minimally as compared to MPC with external dithering, which resulted in excessive perturbations of the process. Figures 9 and 10 compare the process input and controlled output for these two cases. It is clear that the variability in the process output using MPCI is much less than that obtained using MPC with ad hoc external dithering. Thus, this example demonstrates the applicability of the MPCI scheme to update process models with minimal control performance deterioration.

5. CONCLUSIONS

In this work, we formulated a new approach to simultaneous constrained Model Predictive Control and Identification (MPCI). The proposed approach relies on the development of a PE criterion for processes described by DARX models. That PE criterion is used as an additional constraint in the standard on-line optimization of MPC. The resulting on-line optimization problem of MPCI is handled by solving a series of semi-definite programming problems. The advantages of MPCI in comparison to other closed-loop identification methods are

- Constraints on process inputs and outputs are handled explicitly;
- Deterioration of output regulation is kept to a minimum, while closed-loop identification is performed.

While the applicability of MPCI was clearly illustrated by a number of simulation studies, future investigation is needed for a number of theoretical and computational issues, including the following:

- Asymptotic convergence of model parameters.
- Convergence of the MPCI on-line optimization to the global optimum.
- Tuning of MPCI, i.e. selection of values for the parameters that appear in the formulation of the on-line optimization problem.

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Figure Captions

Figure 1: Process input/output for MPCI in *Case Study A*.

Figure 2: Estimates of the drifting parameter in *Case Study A.*

Figure 3: Output and Input plots for Case I in *Case Study B*.

Figure 4: Identified parameters for Case I in *Case Study B*.

Figure 5: Output and Input plots for Case II in *Case Study B*.

Figure 6: Identified parameters for Case II in *Case Study B*.

Figure 7: Output and Input plots for Case III in *Case Study B*.

Figure 8: Identified parameters for Case III in *Case Study B*.

Figure 9: Process inputs for the CSTR using MPCI and MPC with dithering in *Case Study C*.

Figure 10: Process outputs for the CSTR using MPCI and MPC with dithering in *Case Study C*.

Figure 11: Parameter and disturbance estimates in *Case Study C* with adaptive MPC without dithering.

Figure 12: Disturbance rejection by the different model predictive controllers in *Case Study C*.

Table Captions

Table 1: Tuning parameters used in *Case Study A*.

Table 2: Parameter values and input bounds in *Case Study B*.

Table 3: Mean input-output and input-output mean square deviations in *Case Study B*

Table 4: Parameters, inlet Conditions, and steady state values for the CSTR in *Case Study C*.

Table 5: Controller Parameters used in MPCI for the CSTR in *Case Study C*.

Table 1

MPCI Tuning Parameters	**Value**
Horizon Length (*M*)	7
Move Suppression (r_i)	0.1
Forgetting Factor (λ)	1.0
q	100
Excitation Level (γ_1)	0.1
Output Weighting (w_i)	1.0
Identification Window (s)	10

Table 2

	Case I	Case II	Case III
a	0.6	0.6	0.6
b	0.2	0.2	0.2 for $k < 25$ $0.2 - 0.00133(k - 25)$ for $25 \le k \le 75$ 0.1333 for $k > 75$
u_{max} (= $-u_{min}$)	1.0	0.6	1.0

Table 3

	Case I	**Case II**
Mean Output Value	0.1954	0.1167
Output deviation from setpoint	7.13×10^{-4}	1.8×10^{-3}
Mean Input Value	0.3923	0.2351
Input deviation from 0	0.0119	0.0400

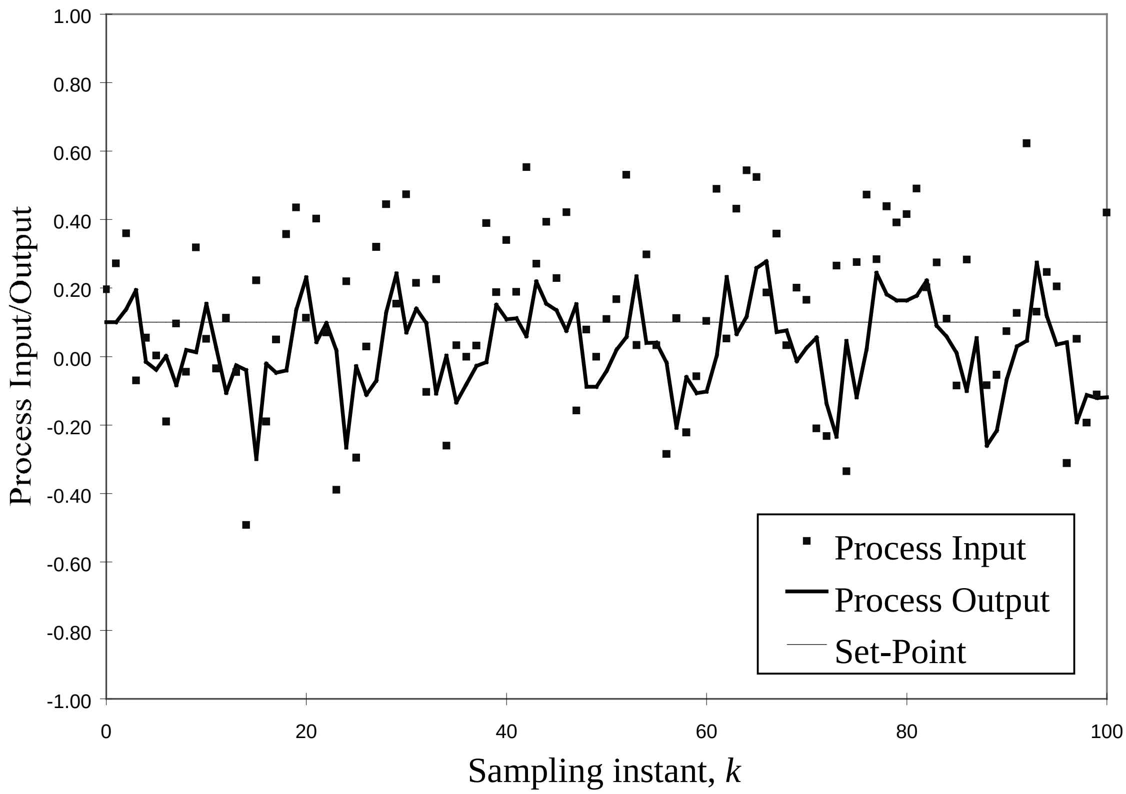
Table 4

F	1.133 m^3/h
V	1.36 m^3
C_{Ai}	8,008 mol/m^3
K	7.0e7 h^{-1}
E/R	8,375 K
T_i	373.3 K
C_p	3,140 $J/Kg/K$
$(-\Delta H)$	69,775 J/mol
C_p	3,140 $J/Kg/K$
ρ	800.8 kg/m^3
UA_t	7.406e6 $J/K/h$
T_s	547.6 K
C_{As}	393.2 mol/m^3
T^s_c	532.582 K

Table 5

Horizon, M	7	Sampling Time, T_s	0.08 h
Move Suppression, r_i	0.8	Output weighting W_I	0.8
Excitation level γ_1	1.0	q	100
Initial Covariance P_o	$10^6\mathbf{I}$	RLS parameter α	0.95

Figure 1



1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
Process Input/Output
0
20
40
60
80
100
Sampling instant, *k*
Process Input
Process Output
Set-Point

Figure 2

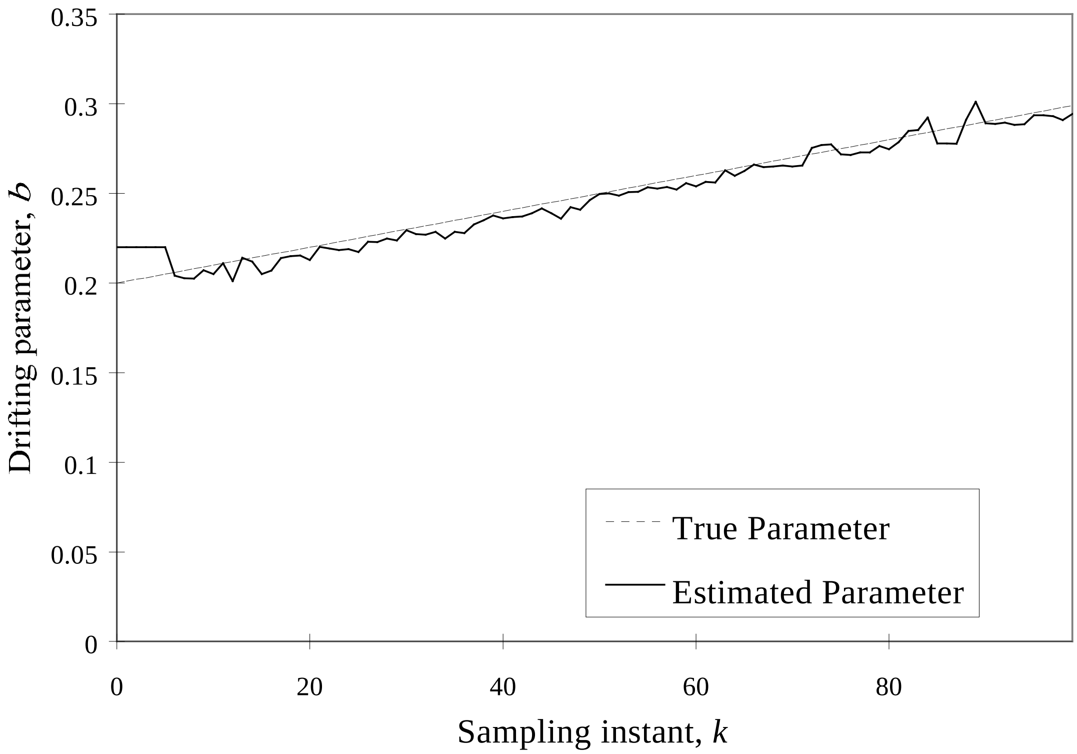

0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Drifting parameter, b
True Parameter
Estimated Parameter
0
20
40
60
80
Sampling instant, k

Figure 3

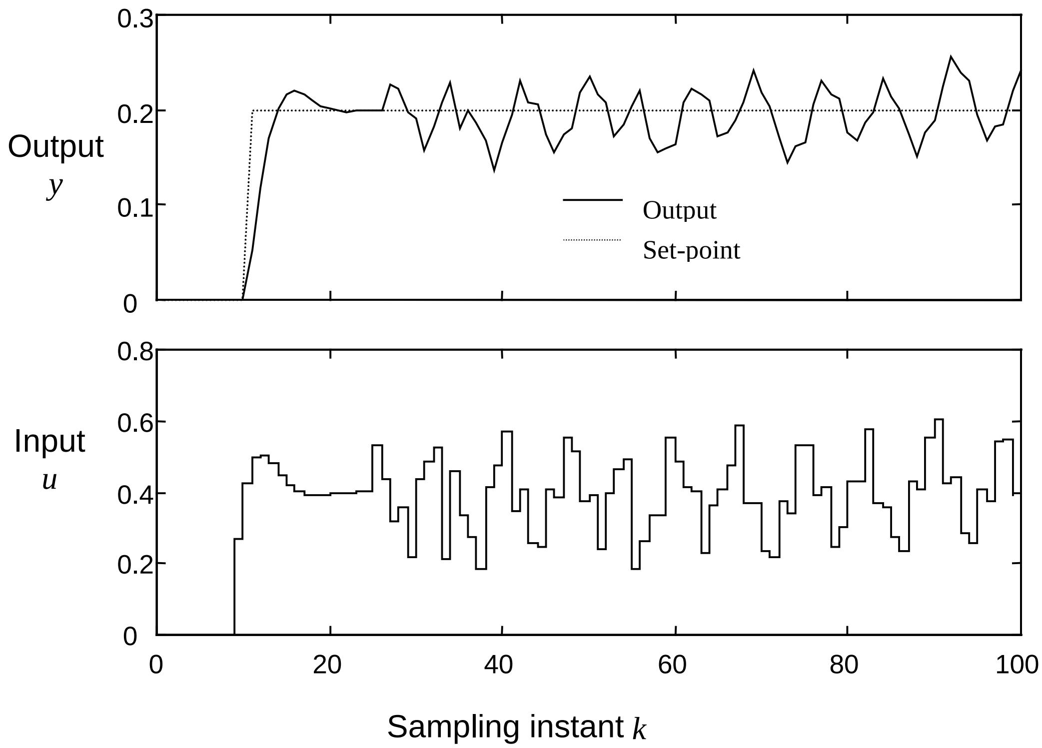

0.3
0.2
0.1
0
Output
y
Output
Set-point
0.8
0.6
0.4
0.2
0
Input
u
0
20
40
60
80
100
Sampling instant k

Figure 4

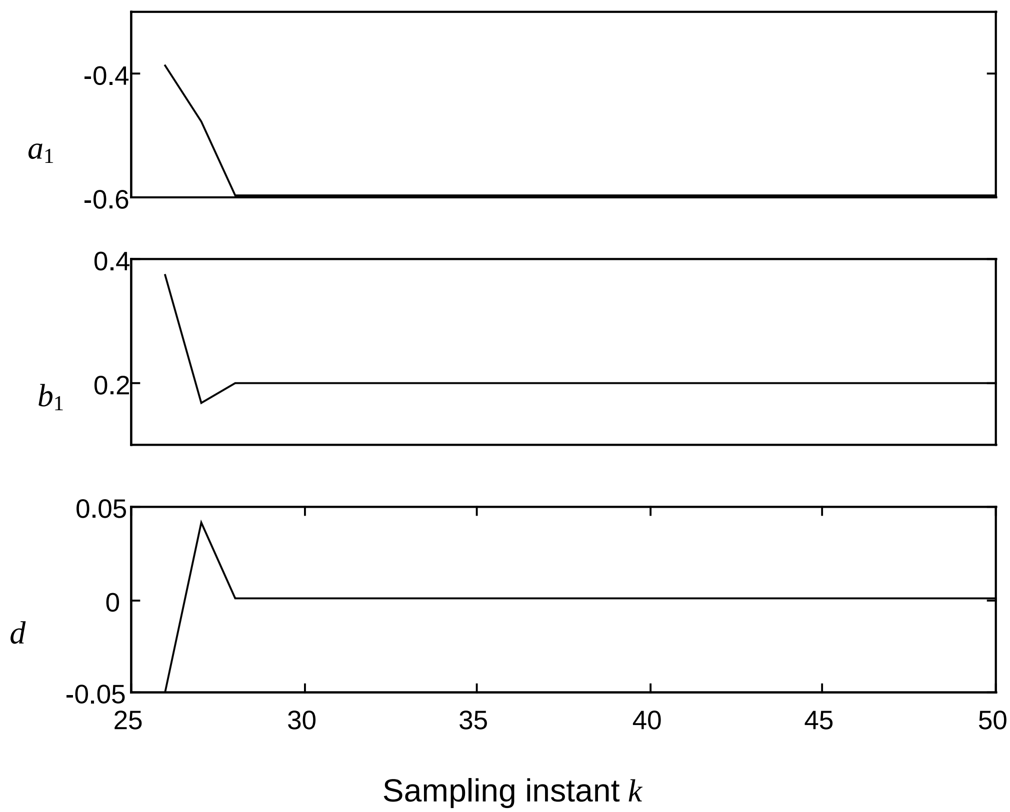
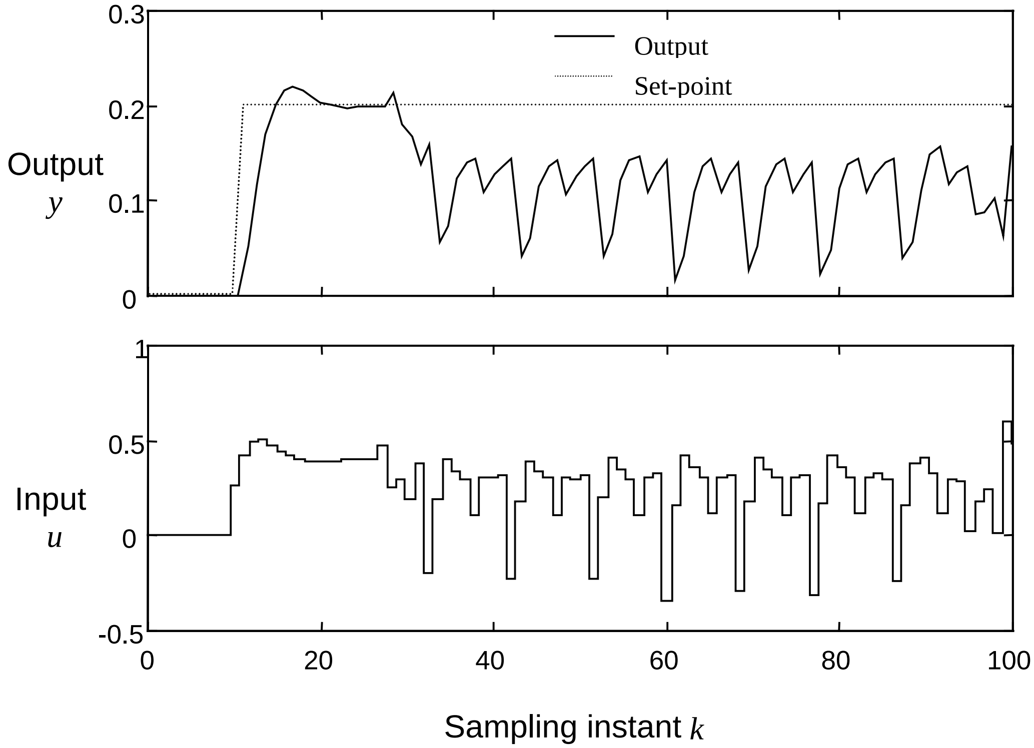


Figure 5



0.3
0.2
0.1
0
Output
Set-point
Output
y
1
0.5
0
-0.5
Input
u
0
20
40
60
80
100
Sampling instant k

Figure 6

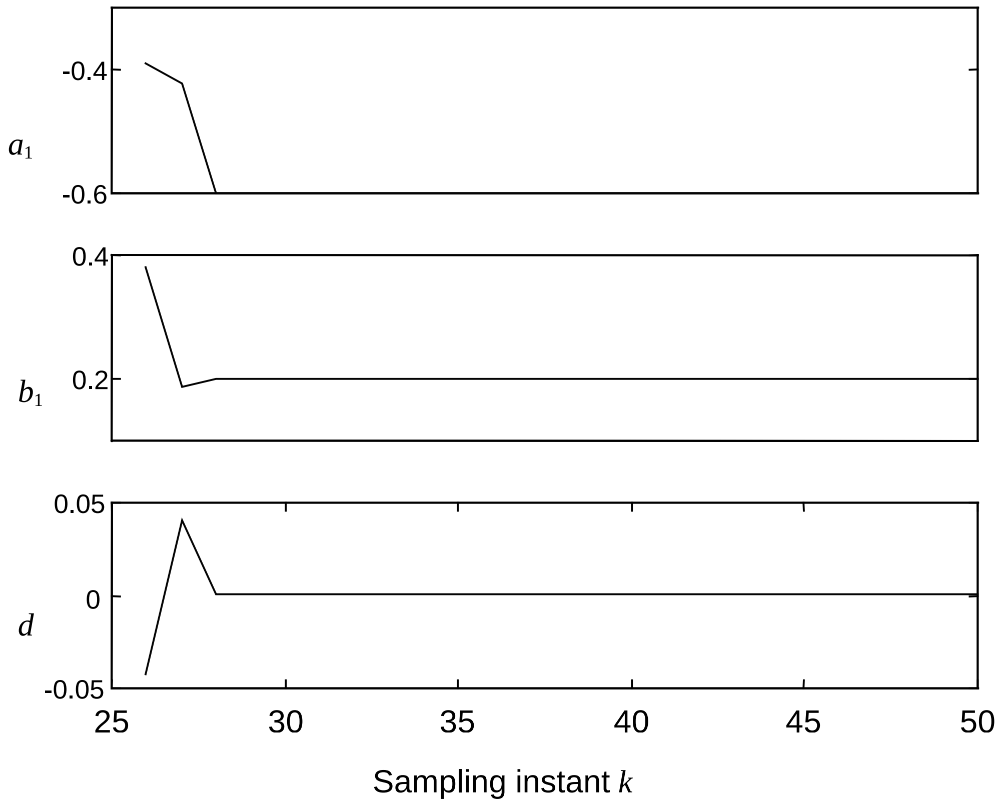
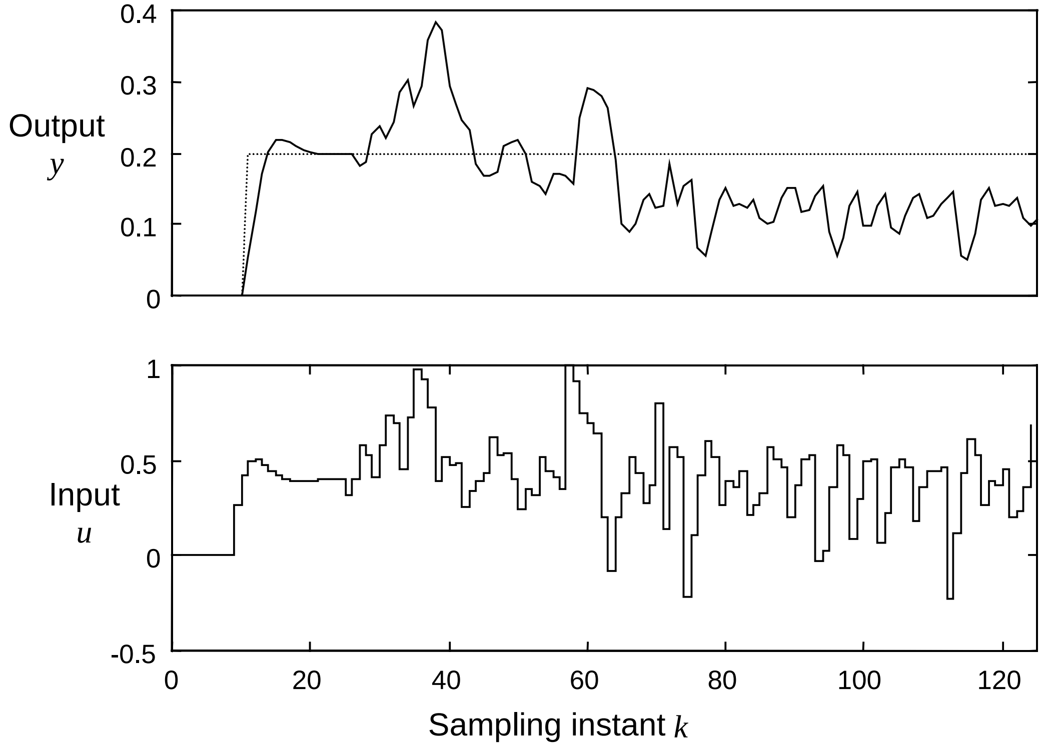
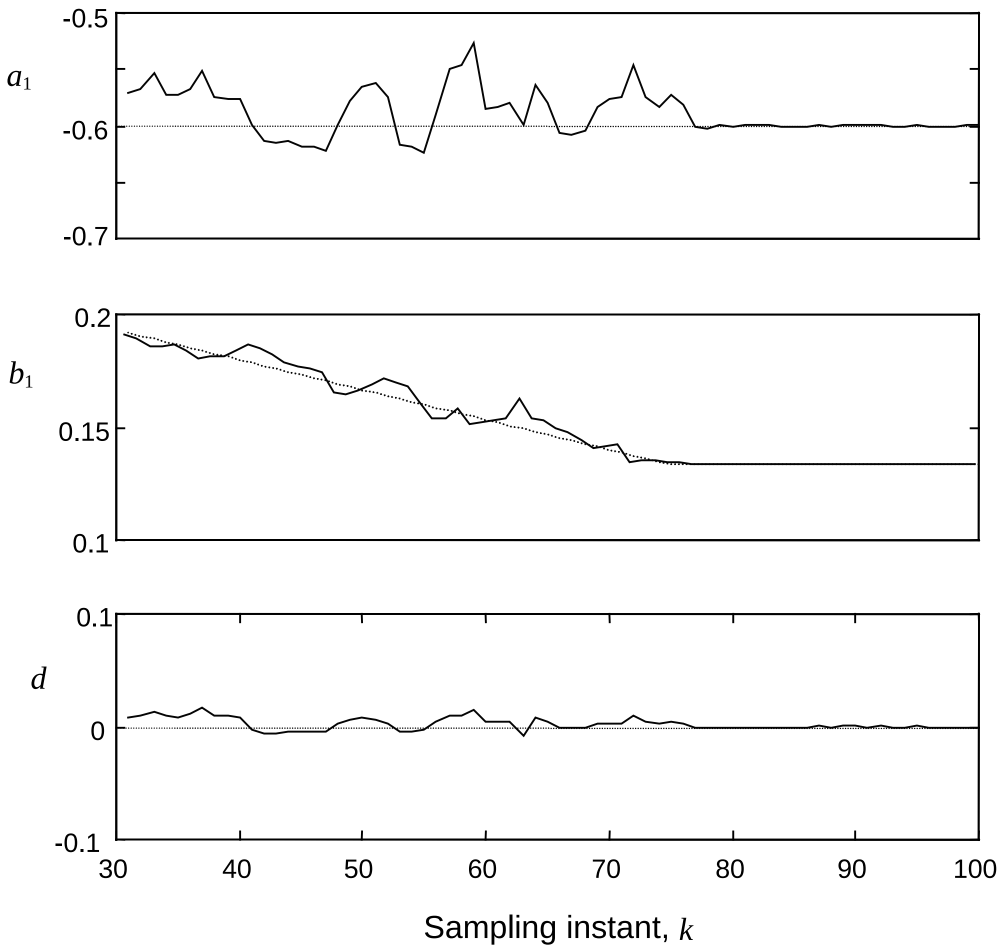


Figure 7



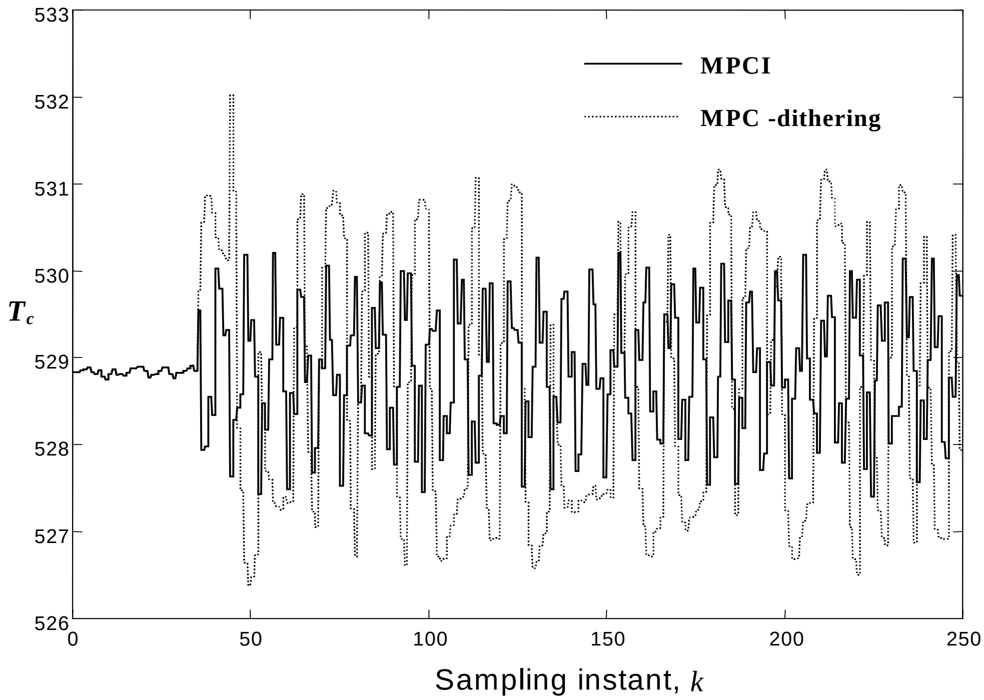
0.4
0.3
0.2
0.1
0
Output
y
1
0.5
0
-0.5
Input
u
0
20
40
60
80
100
120
Sampling instant k

Figure 8



-0.5
a_1
-0.6
-0.7
0.2
b_1
0.15
0.1
0.1
d
0
-0.1
30
40
50
60
70
80
90
100
Sampling instant, k

Figure 9



MPCI
MPC -dithering
533
532
531
530
529
528
527
526
T_c
0
50
100
150
200
250
Sampling instant, k

Figure 10

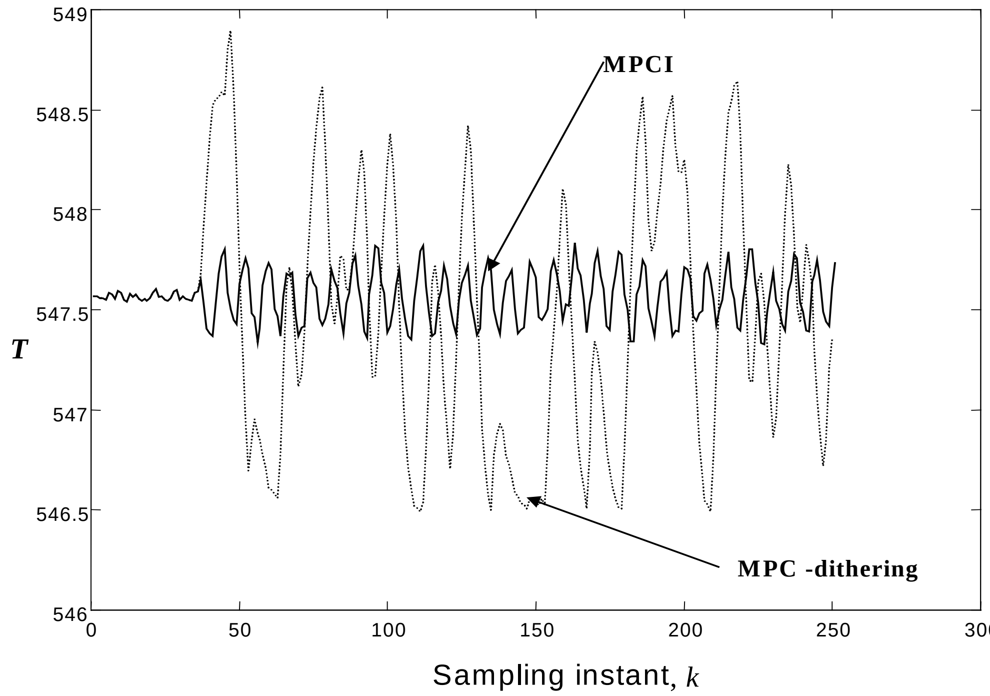

MPCI
MPC -dithering
549
548.5
548
547.5
547
546.5
546
T
0
50
100
150
200
250
300
Sampling instant, k

Figure 11

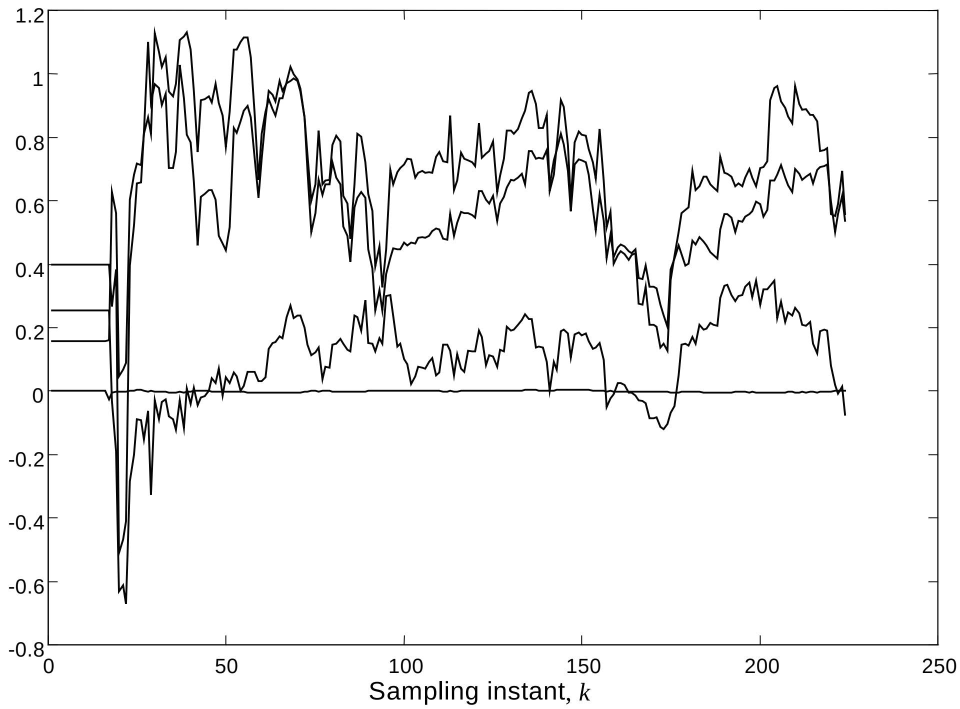
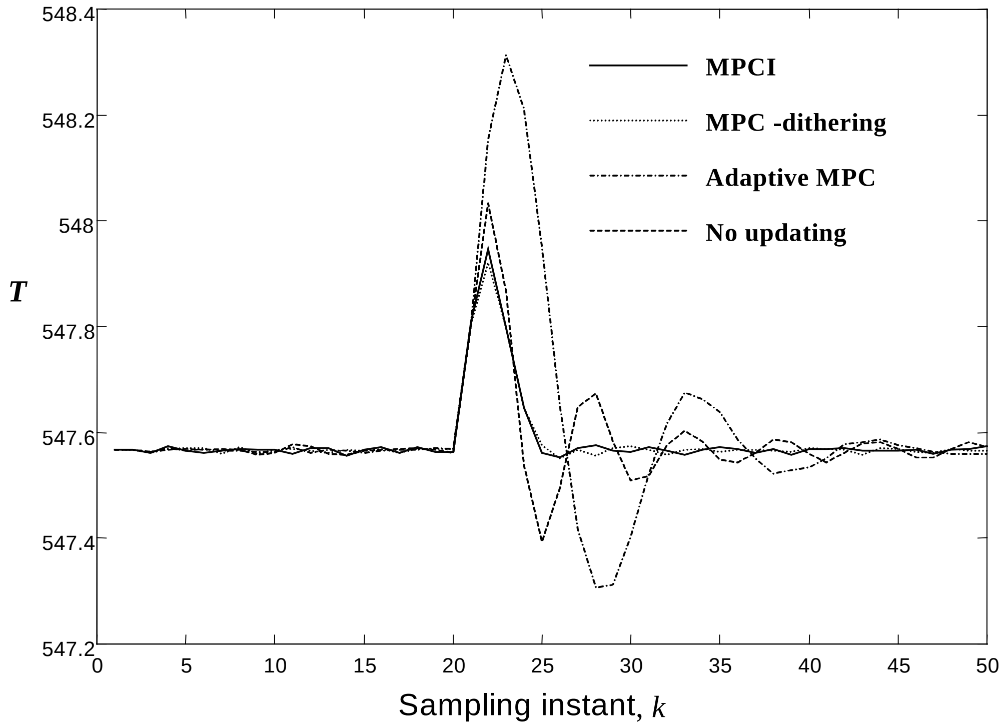

1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
50
100
150
200
250
Sampling instant, *k*

FIGURE 12



548.4
548.2
548
547.8
547.6
547.4
547.2
T
0
5
10
15
20
25
30
35
40
45
50
Sampling instant, k
MPCI
MPC -dithering
Adaptive MPC
No updating